Global Conference on Multidisciplinary Research and Innovation
Hosted Online from Berlin, Germany
Date: 2nd June, 2026
Website: https://econferencia.com

USING THE METHOD OF MATHEMATICAL INDUCTION IN COMBINATORICS

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Abstract

In practice, it is often necessary to select a subset of elements with certain properties from a given set of objects, to arrange the elements of one or more sets in a specific order, and to study similar problems.

Since such problems involve various combinations of objects, they are called combinatorics problems, and the branch of mathematics that studies combinatorics problems is called combinatorics.

Suppose a set consisting of n elements is given. When creating different groups from the elements of this set, it is important to answer whether the order of the selected elements matters or not, and whether the same elements can be repeated or not.

We present some combinatorics problems that answer the questions above and can be proven using mathematical induction.

Definition of permutations (arrangements): Permutations of n elements taken m at a time $(n \geq m)$ are combinations that contain m elements each. One combination differs from another either by the composition or the order of its elements.

Example 1. For the number of permutations, the formula

$$A_n^m = n(n-1)...(n-m+1) \qquad (1)$$

holds, where $m = \overline{1,n}$

Proof. I. For $m = 1$, $A_m^1 = n$, so (1) holds.

II. Let us assume that for $m = k$ $(k < n)$

$$A_n^k = n(n-1)...(n-k+1)$$

holds. Using this assumption, we will show that formula (1) is also valid for $m = k+1$.

To form permutations of n elements taken $k+1$ at a time, it is sufficient to append one of the remaining $(n-k)$ elements to the end of each permutation of n elements taken k at a time. Therefore,

$$A_n^{k+1} = A_n^k(n-k) = n(n-1)...(n-k+1)(n-k)$$

Definition of permutations (rearrangements): Permutations of n elements are combinations that contain all n given elements, and they differ from one another only by the order of their elements.

Example 2. For the number of permutations of n elements, the following formula holds:

$$P_n = n! \qquad (2)$$

Proof. I. For $n = 1$, $P_1 = 1$, so (2) holds.

II. Let (2) be valid for $n = k$, that is, $P_k = k!$. Using the inductive hypothesis, we will show that formula (2) is valid for $n = k+1$.

The number of possible permutations formed from the given k elements $a_1, a_2, ..., a_k$ is $k!$. Taking one of them, we insert the element a_{k+1} sequentially before the 1st term, 2nd term, ..., k-th term, and after the k - th term of this permutation. Thus, from a permutation of k elements, we generate $k+1$ permutations. Repeating this process for each of the $k!$ permutations, we obtain a total of

$$k!(k+1)=(k+1)!$$

permutations from $k+1$ elements.

Thus, all the generated permutations are distinct, which implies that the number of all possible permutations formed from $k+1$ elements is

$$P_{k+1}=(k+1)!$$

Definition of combinations: Combinations are selections where the order of elements does not matter. One combination differs from another by at least one element.

Example 3. For the number of combinations of n elements taken m at a time, the following formula holds:

$$C_n^m=\frac{n(n-1)...(n-m+1)}{1\cdot 2\cdot 3\cdot ...\cdot m} \qquad (3)$$

Proof. I. For $m=1$, $C_n^1=n$, meaning (3) holds.

II. Let us assume that for $m=k$

$$C_n^k=n(n-1)...\frac{[n-(k-1)]}{1\cdot 2\cdot ...\cdot k}$$

holds. We will prove that formula (3) is valid for $m=k+1 \quad (k+1)\le n$.

If we take all combinations of n elements taken k at a time and append each of the remaining $n-k$ elements to each combination as the $(k+1)$-th element, it is clear that all possible combinations of n elements will be obtained in this way, but each of them will be generated $k+1$ times. Indeed, the combination $a_1,a_2,...a_{k+1}$ is formed by appending a_1 to $a_2,a_3,...,a_k,a_{k+1}$, appending a_2 to $a_1,a_3,...,a_k,a_{k+1}$, and

so on, and appending a_{k+1} to $a_1, a_2, ..., a_k$. Since they must differ by their elements, only one of them should be taken. Thus,

$$C_n^{k+1} = C_n^k \cdot \frac{n-k}{k+1} = \frac{n(n-1)...(n-k)}{1 \cdot 2 \cdot 3 \cdot ... \cdot (k+1)}$$

From the last formula, it follows that (7.3) is valid for $0 \le m \le n$.

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